2. Recall that many bosons (Cooper pairs) may occupy the ground state.

Identify superconducting current with that carried by Cooper pairs.

Normal current carried by single electrons (fermions) in excited states.

Ground state separated from excited states by energy gap Δ.

Pairs have range of interaction - called coherence length ($\sim 10^{-6}$ m)

3. Q. Is the superconducting current a Bose-Einstein condensation of Cooper pairs?

A. Large numbers of Cooper pairs can occupy the ground state and cause superconducting current

Wavefunction of pairs is correlated – single w/f for all pairs as in Bose – Einstein condensate

But evaluation of Bose-Einstein condensation temperature T_B using actual electron density $\left(\frac{N}{V}\right)$ gives

$T_B \gg T_c$

Thus:

(i) Superconductivity depends on a condensation of bosons (Cooper pairs) into ground state

(173)

(ii) Critical temperature T_c set by strength of pairing interaction not by Bose - Einstein condensation

Further points.

4. Magnetic field acts to destroy Superconductivity because its action is to align electron spins $\parallel B$ – tending to break up $\uparrow\downarrow$ pairs

5. BCS theory relates energy gap $\Delta(0)$ and electron - lattice interaction strength V via

$$\Delta(0) = 1.76\, kT_c = 2\hbar\omega_D \exp\left(-\frac{1}{g(\mu).V}\right)$$

where $\omega_D = \dfrac{k\Theta_D}{\hbar}$ ← Debye temperature

and $g(\mu)$ is density of states at Fermi surface

6. Isotope effect — first evidence that lattice concerned in pairing interaction

For superconductors Hg, Pb, Sn, Zn ..

Value of T_c for samples of different isotopes of same element related to isotope mass M by

$$T_c \propto \left(\frac{1}{M}\right)^{1/2}$$

Follows from $T_c \propto \omega_D \propto \left(\frac{K}{M}\right)^{1/2}$

where K is bond constant.

M is isotopic mass.

7. Experiment shows energy gap Δ effectively constant over range

$0 < T < T_c/2$

For temperature range $\frac{T_c}{2} < T < T_c$

approximate relation

$$\Delta(T) \simeq 3.1\, k T_c \left[1 - \frac{T}{T_c} \right]^{1/2}$$

— see sheet.

8. Seen above that —

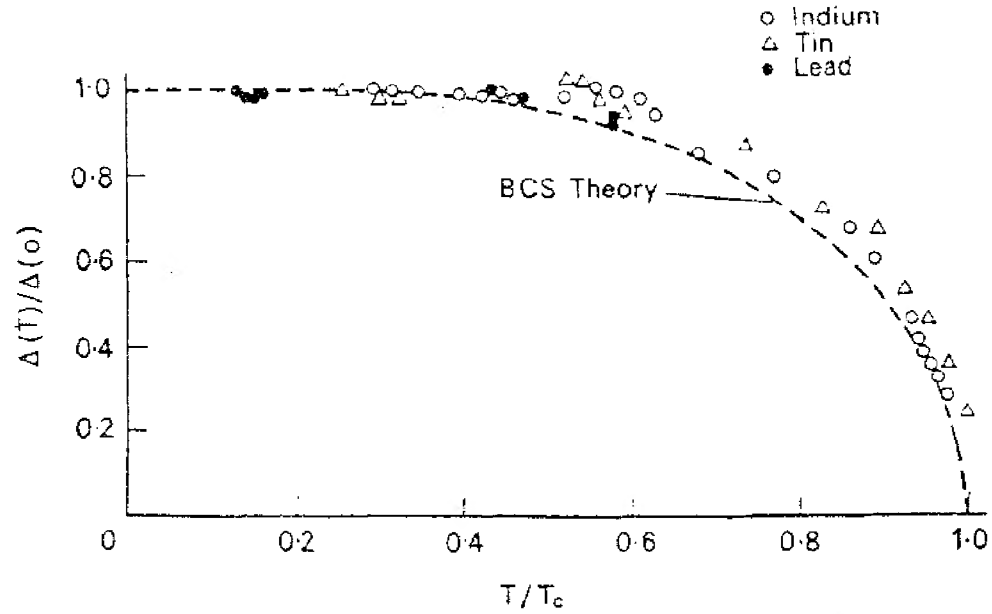
Large electron lattice interaction V related to high T_c

In normal state large V means large (for metal) resistivity

Connection between superconductivity and poor metallic conductivity in normal state

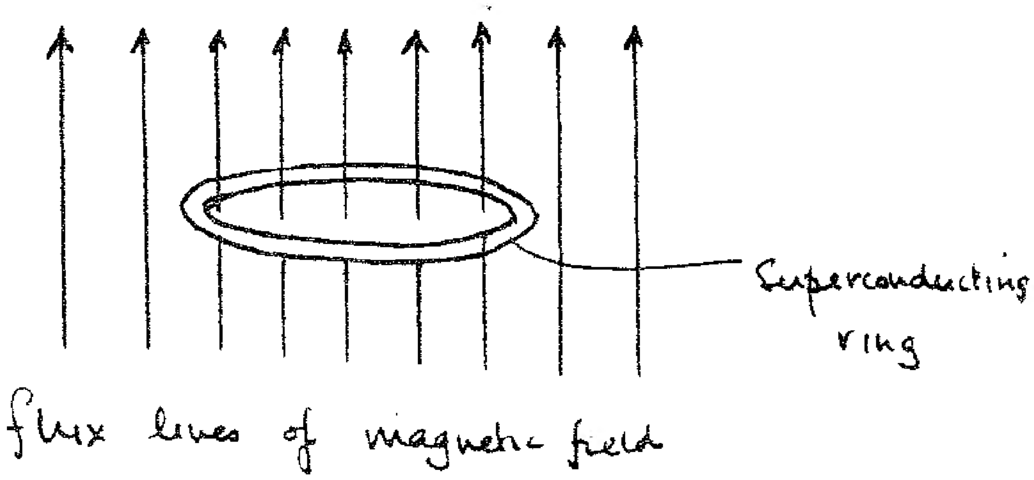
Reason why good conductors Cu, Ag, Au .. do not show superconductivity.

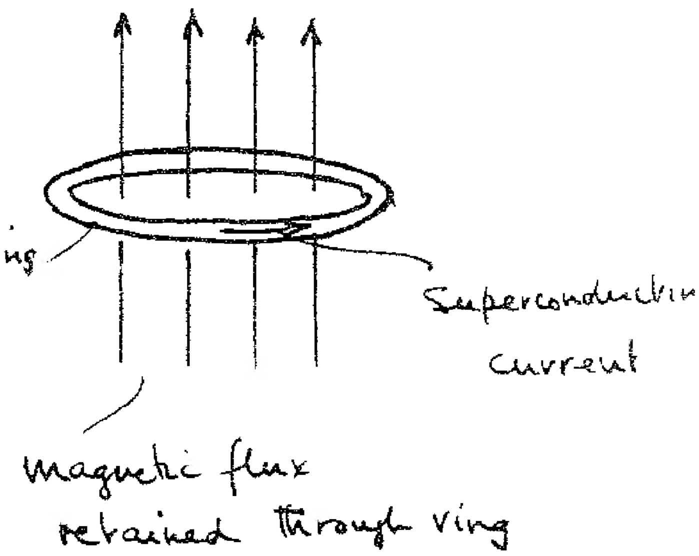
Variation of energy gap $\Delta(T)$ with temperature.

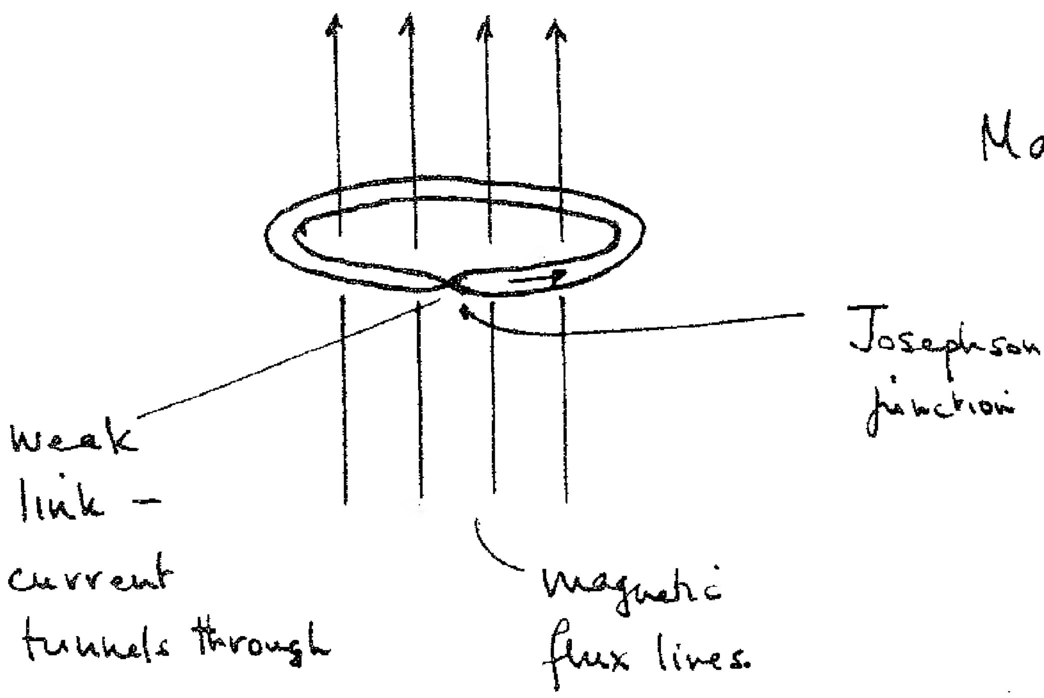


Magnetic field on

Field turned off







Magnetic field $B = \dfrac{\text{Flux } \phi}{\text{Area } A}$.

Flux quantisation through a superconducting ring

Schematic diagrams — see sheet.

When magnetic field removed — magnetic flux passing through hole in the ring is maintained by superconducting current flowing around ring.

Can be shown that total flux through ring ϕ is

$$\phi = n\phi_0$$

where n = integer

ϕ_0 = unit of flux $= \dfrac{h}{2e}$ Tm^2

Create weak link in ring — Josephson junction — flux can escape — one line at a time — see sheet.

By inductive coupling of coils to Josephson loop — get signal for each ϕ_0 that enters or escapes from loop.

Basis of SQUID

(Superconducting quantum interference device)

– most sensitive method of measuring magnetic field.

BCS prediction

$$\phi_0 = \frac{h}{q}$$

where q is charge of current carrier

Measured value $\phi_0 = \frac{h}{2e}$

gives $q = 2e$

confirms superconducting current carried by electron pairs.

Type I and Type II superconductors.

Different M vs H graphs - see sheet.

Type I - for $0 < H < H_c$ all flux excluded from superconducting cylinder

Type II - for $0 < H < H_{c_1}$ same as Type I

for $H_{c_1} < H < H_{c_2}$ flux line penetration (mixed state

for $H_{c_2} < H$ normal state

Penetrating flux quanta in mixed state minimise energy by equalising their separation - creates flux line lattice - see sheet.

In practice impurities in sample can pin flux lines

Type II superconductors more practical in high field uses – magnet coils – since usually H_{c_2} > Type I H_c.

Parallel between flux line penetration of Type II superconductor and quantised vortices (rotons) in liquid He^4 II.

New superconducting materials.

For elements highest $T_c = 9.5\,K$ Nb.

For alloys " $T_c = 23.2\,K$ Nb_3Ge

Many attemps to find superconductor with higher T_c (and H_c).

In 1987 – new type of oxide (ceramic)

superconductors found with

higher values of T_c

Values	T_c
$La_{1.85}Sr_{0.15}CuO_4$	36 K
$YBa_2Cu_3O_7$	90 K
$Tl_2Ba_2Ca_2Cu_3O_{10}$	120 K

Materials have crystal structure of Cu-O planes spaced apart by other atoms – see sheet.

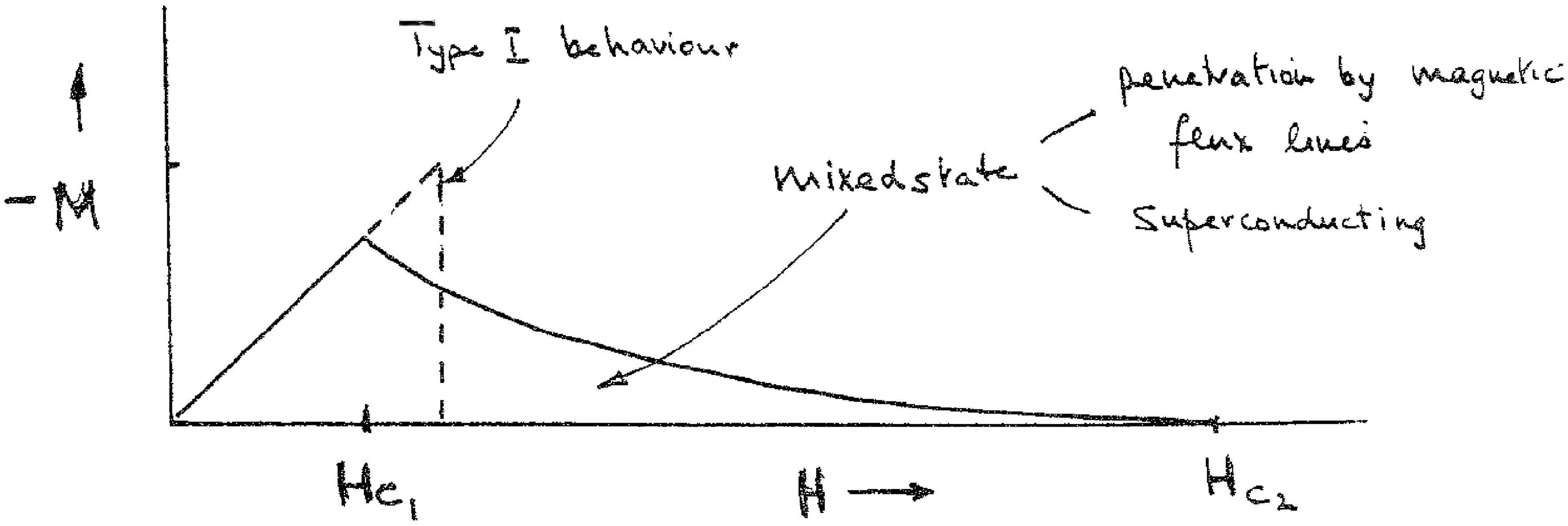
Superconductivity occurs mainly along Cu-O planes

High T_c means strong pairing interaction

Mechanism for this still not clear

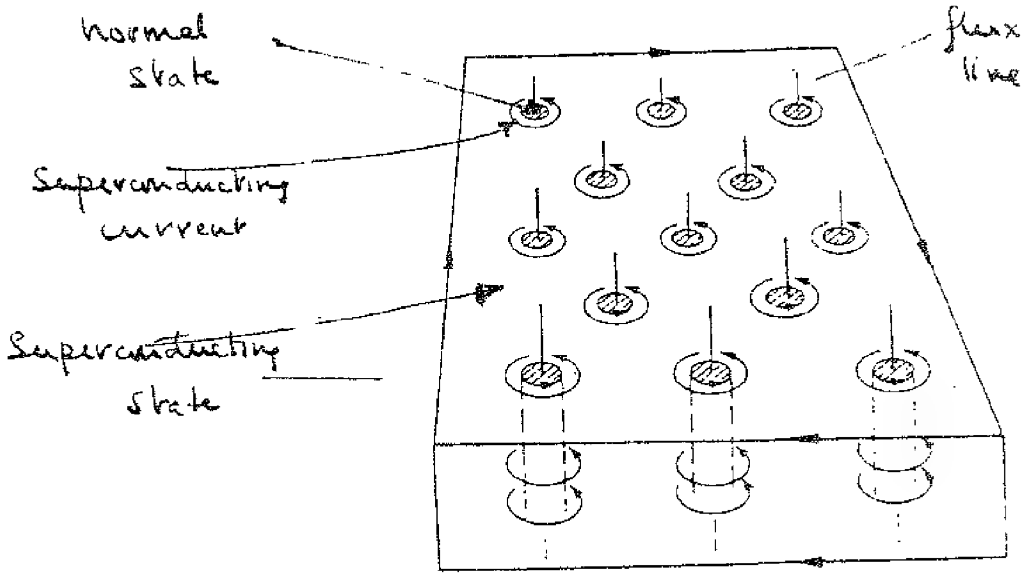
Materials are type $\underline{\text{II}}$ with large H_{c_2} but so far have been unable to pass large currents without becoming normal.

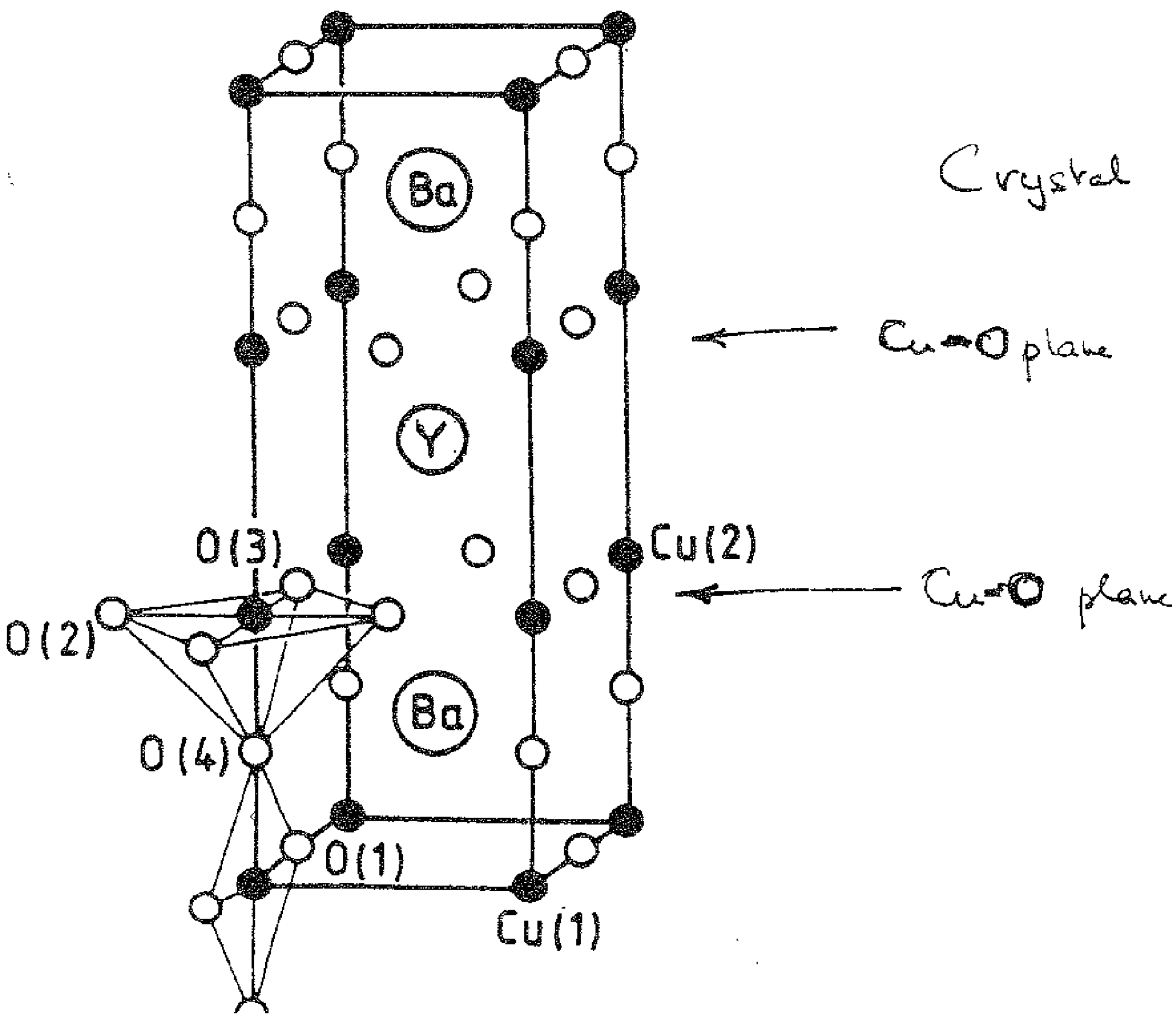
Type II Superconductors



Picture of mixed state of Type II Superconductor.

Note lattice (pattern) of flux lines





Crystal structure of $YBa_2Cu_3O_7$ showing Cu-O planes

Superfluid He^3

Seen that superconducting behaviour arises from Bose-Einstein condensation where bosons are coupled pairs of electrons

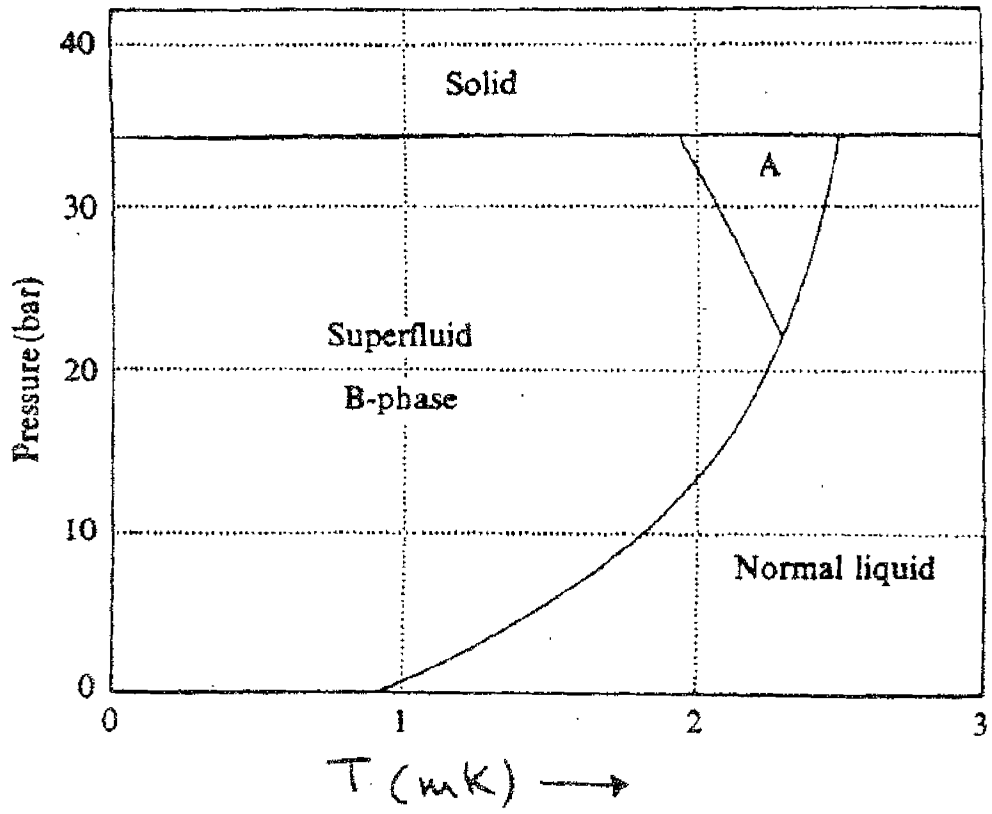
Question. Can He^3 atoms (fermions) pair to form bosons allowing a Bose-Einstein condensation and superfluid properties?

Ans. Yes - but at low temps $\lesssim 2mK$.

Low temperature phase diagram of He^3

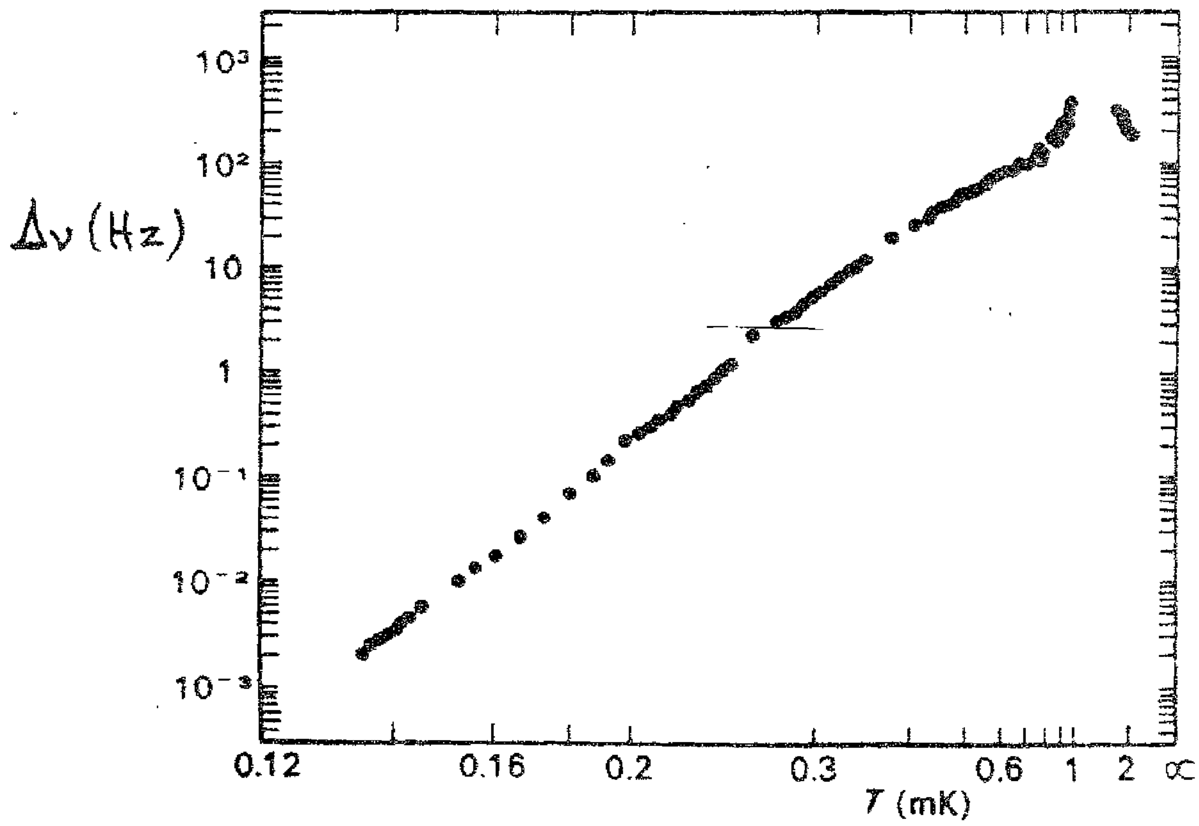
— See sheet.

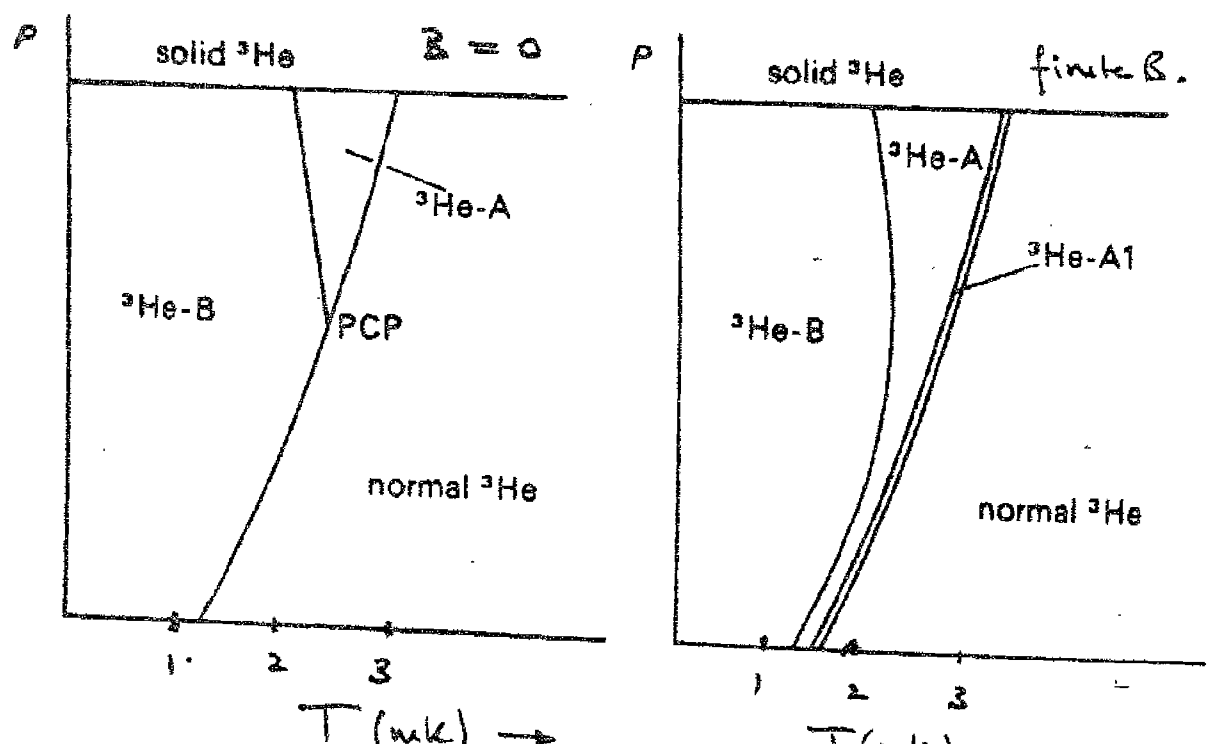
Superfluid He^3



Low temperature phase diagram

Damping of oscillating wire in liquid He^3





Effect of magnetic field on phase diagram

Diagram shows for $T \leq 2\,mK$ two different superfluid phases – phase A
phase B.

Properties of these phases complex – just discuss main points.

Points.

1. Proof of superfluid nature.

Expt – damping of wire vibrating at freq ν in liquid He^3.

Stimulate vibration of wire – resonance at natural freq ν

Measure width of resonance $\Delta\nu$

$\Delta\nu$ depends on damping force (viscosity) of liquid He^3

Plot of $\Delta\nu$ versus T — see sheet.

Damping force decreases by 10^5 between 1.04 mK and 0.14 mK.

2. Viscosity expts (like above) consistent with 2 fluid model where fluid normal at critical temp T_c but proportion of superfluid increases as temp decreased

All superfluid as $T \to 0$ K.

Superfluid seen as He^3 pairs in g/state

Normal fluid " He^3 single atoms in excited states

As in superconductors – strength of pairing interaction determines critical temp T_c

3. Mechanism of pairing. — not phonons

but type of magnetic interaction.

Atom 1 polarises surrounding He^3 atoms

Atom 2 interacts with polarisation trail.

Details complex.

Interaction weak — since $T_c \sim 2\,mK$.

4. Character of pairs

In superconductors — electron pairs have

$L = 0$, $S = 0$

Superfluid He^3 pairs shown (by expt)

to have $S = 1$

Then Pauli principle (total w/f of pair

antisymmetric with

particle exchange)

requires that orbital ang mtm of pair must be L = odd integer

actually $L = 1$.

Thus He^3 superfluid pairs have $L = 1$, $S = 1$.

From quantum mechanics.

$S = 1$ has substates

$S_z = +1 \quad \uparrow\uparrow$

$S_z = 0 \quad \frac{1}{\sqrt{2}}\{\uparrow\downarrow + \downarrow\uparrow\}$

$S_z = -1 \quad \downarrow\downarrow$

Distinction between

Phase A contains $S_z = +1 \quad \uparrow\uparrow$

and $S_z = -1 \quad \downarrow\downarrow$

pairs only

Phase B contains $S_z = +1, 0, -1$ pairs

5. Effect of magnetic field.

Pairs with $S = 1 \quad S_z = +1$ } have magnetic moment

$S = 1 \quad S_z = -1$

— thus superfluid affected by magnetic field.

Effect of phase diagram on field – see sheet.

Phase A more magnetic than Phase B — since all pairs have magnetic moment

For field $> 0.6T$ — no phase B.

New phase A_1 occurs between normal fluid and Phase A

Phase A_1 thought to contain just

$S_z = +1 \quad \uparrow\uparrow$

or $S_z = -1 \quad \downarrow\downarrow$ pairs

6. Pairs have $\underline{L}$ vector normal to $\underline{S}$ to minimise pair energy.

In phase A

L vector wants to be (i) $\perp$ to walls of container

(ii) $\perp$ to $\underline{S}$

While in magnetic field $\underline{S}$ wants to be $\parallel$ to field.

These requirements ~~usually~~ conflict giving rise to complex, anisotropic effects.

Phase B similarly complex.

Further details

Introduction to Liquid Helium

Wilks and Betts.

Main point.

Superfluid He^3 exists by pairing

of He^3 atoms to form bosons that make Bose-Einstein condensation to ground state.

Practical methods of reaching low temperature

Energy U, Entropy S and Temperature T.

At low temperatures it is more helpful to associate temperature T with entropy rather than energy.

Recall Entropy $S = k \ln \Omega$

where Ω = number of microstates of system

Qualitatively S represents the disorder in the system

- in an equlibrium system as $T \to 0$
  system goes into ground state $\Omega = 1$
  and $S = 0$
  but energy U is not necessarily $= 0$.

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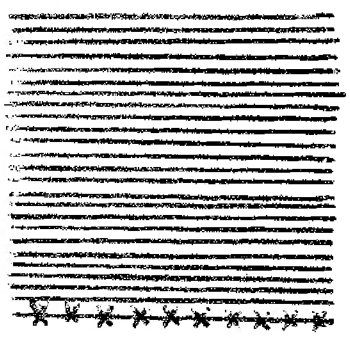
I Illustrations

(i) System of N bosons.

As $T \to 0$

Bosons condense into ground state



giving

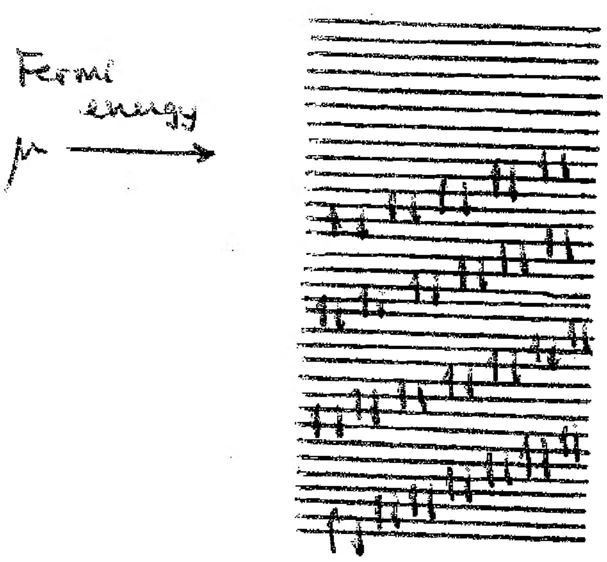
$U \to 0$ (not counting quantum zero point energy)

$S \to 0$

(ii) System of N electrons (fermions)

As $T \to 0$

Electrons condense to lowest set of states – but Pauli principle does not allow two electrons in same quantum state



As $T \to 0$

Energy $U \to \frac{3}{5} N\mu$

$S \to 0$